Slide 1

Learning Trajectories of Early Math

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Slide 2

Learning Trajectories

- Mathematics *of* children— *representations* and thinking of children as it developments naturally

- Activities *matched* to children's development in each topic

- Therefore:

  - All within developmental capacities of children

  - Provide a natural "building block" to the next level

  - Provides *mathematical* building blocks for school success, because research based on typical higher-income children
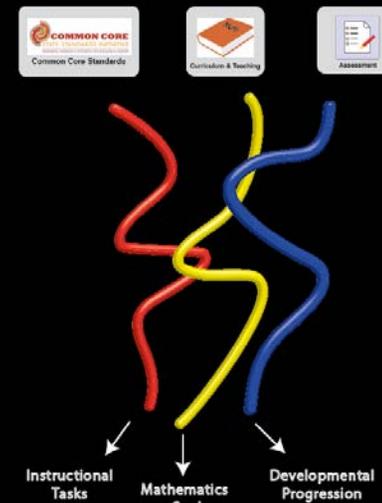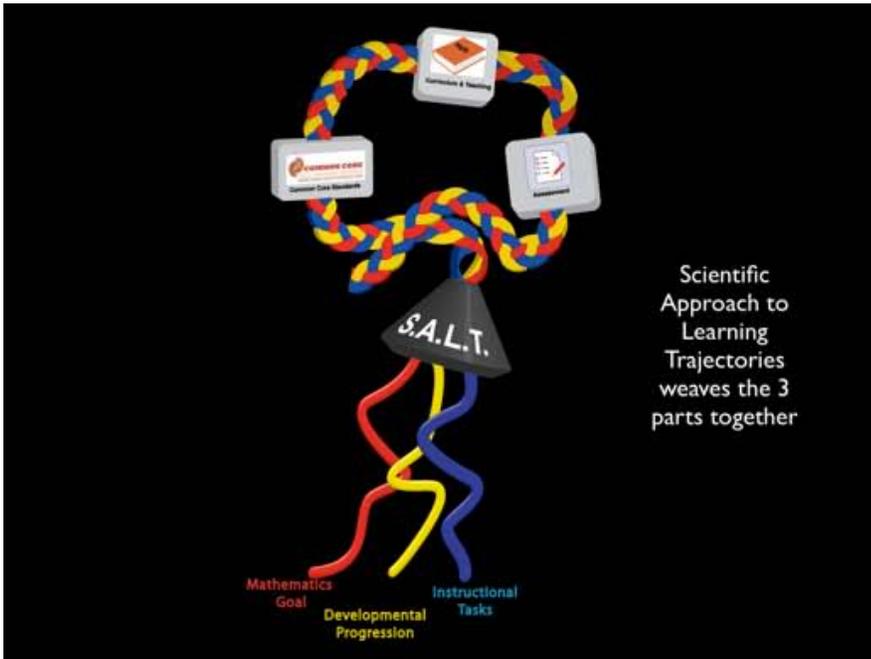
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Learning Trajectories:
3 Parts

1. Goal

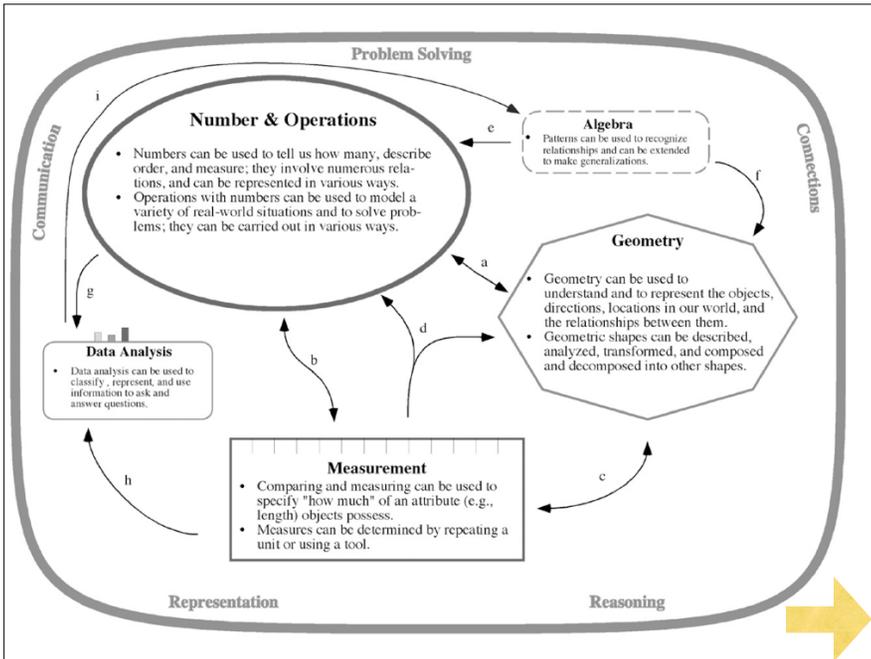2. Developmental Progression

3. Instructional Activities

learning trajectories

developmental path
For each topic, such as "counting," research and practice have helped identify levels of thinking through which children move as they learn about that topic. For example, they move from a level in which they can only count out loud, to counting objects accurately, to ever more sophisticated counting strategies.

instructional path
The activities are designed to help children move along the developmental path. For example, children who can count out loud are engaged in activities that help them learn to count a collection of objects and understand that this helps determined the number in the collection.

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Instructional Tasks   Mathematics Goal   Developmental Progression

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Scientific Approach to Learning Trajectories weaves the 3 parts together

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Conference on Early Math Standards

ExxonMobil Foundation

UNIVERSITY AT BUFFALO
Graduate School of Education

Douglas H. Clements

Julie Sarama

University at Buffalo

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Research-based "Big Ideas"

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Big Ideas

- Research: Teachers who study and focus on big ideas more successful

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Problem Solving

Number & Operations
- Numbers can be used to tell us how many, describe order, and measure; they involve numerous relations, and can be represented in various ways.
- Operations with numbers can be used to model a variety of real-world situations and to solve problems; they can be carried out in various ways.

Algebra
Patterns can be used to recognize relationships and can be extended to make generalizations.

Geometry
- Geometry can be used to understand and to represent the objects, directions, locations in our world, and the relationships between them.
- Geometric shapes can be described, analyzed, transformed, and composed and decomposed into other shapes.

Data Analysis
- Data analysis can be used to classify, represent, and use information to ask and answer questions.

Measurement
- Comparing and measuring can be used to specify "how much" of an attribute (e.g., length) objects possess.
- Measures can be determined by repeating a unit or using a tool.

Communication · Connections · Representation · Reasoning

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

What Might Be Missed

- Learning Trajectories at the *Core* of the Common Core

Learning Trajectory for Counting

1st: Goal: Accurate, confident object counting

Learning Trajectory for Counting

1st: Goal: Accurate, confident object counting

2nd: Developmental Progression...

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Learning Trajectory for Counting

- *Precounter* Says number words, not sequence: "one, two, four...".

- *Chanter* Says in sequence but may run together

- *Reciter* Verbal counting to 5, then 10

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Learning Trajectory for Counting

- *Corresponder* Counts correctly using 1-1 correspondence, *at least up to 5 objects in a line.*

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Learning Trajectory for Counting

- *Corresponder* Counts correctly using 1-1 correspondence, *at least up to 5 objects in a line*

- *Counter (Small Numbers)* Counts 1-5 objects in a line meaningfully (i.e., employ the cardinal rule)

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Learning Trajectory for Counting

- *Producer (Small Numbers)* Counts *out* a collection up to 5

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Learning Trajectory for Counting

- *Producer (Small Numbers)* Counts *out* a collection up to 5

- *Counter (10)* Counts collections up to 10

- *Counter and Producer (10+)* and keeps track of unorganized collections

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Learning Trajectory for Counting

- Counter from N

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Learning Trajectory for Counting

- Counter from N

- **Counter On Using Patterns**

- Counter On Keeping Track

- Counter Forward and Back

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Instructional Activities:
3rd Part of Learning Trajectories

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Small Numbers and Counting

- Finger plays:
  - One, two, buckle…
  - When I was one…

    When I was one, I was so small, (hold up 1 finger)

    I could not speak a word at all. (shake head)

    When I was two, I learned to talk. (hold up 2 fingers)

    I learned to sing, I learned to walk. (point to mouth and feet)

    When I was three, I grew and grew. (hold up 3 fingers)

    Now I am four and so are you! (hold up 4 fingers)
  - Later: Five Little Monkeys, etc.

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Count and Move

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Slide 24

Small Group Record Sheet
Building Blocks Math - PreK Assessment

Week: 2
Activity: Find and Make Groups

Trajectory Name	Description
Small Collection Namer	Can name collection of objs up to 3
Maker of Small Collections	Can make objs to 4
Counter (Small Numbers)	Can count objs up to 5/knows "how many?"
Perceptual Subitizer to 4	Can subitize up to 4
Perceptual Subitizer to 5	Can subitize up to 5

	Child's Name	Finds groups to:	Strategies/ Trajectory Level:	Comments:
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				

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Pizza Pizzazz Free Explore
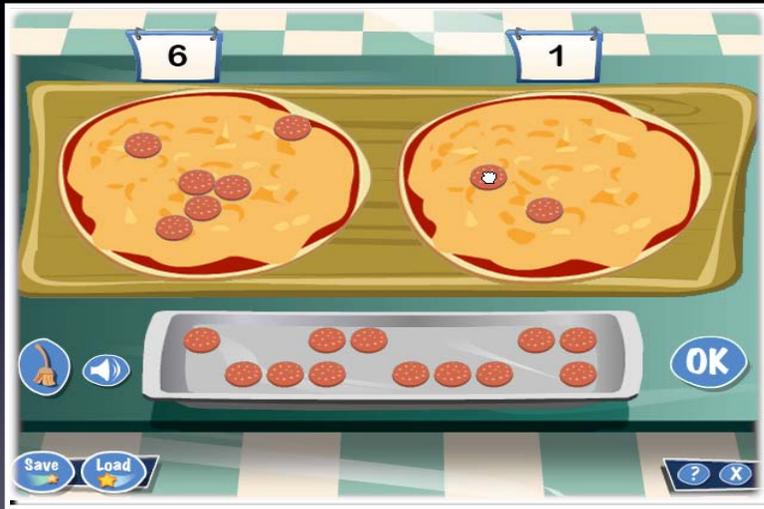
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Activities for multiple goals

- What goals on number table?

  – Make and imagine small collections items *nonverbally*

  – Count by ones to 10

  – Know the last counting word tells "how many"

  – Count out (produce) a collection

  – Subitize (quickly "see" and label with a number)

  – Identify whether collections are the "same" number or which is "more" visually

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Counting Games

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Road Race: Connecting Representations

- Count the dots and move that number of jumps

- *Connecting* different representa-tions of number!

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Road Race Shape Counting - Another Variation

- Count the sides of a shape and move that number of jumps

- Connecting new concepts of number

Road Race Shape Counting

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Space Race Number Choice

- Choose the "better" of two numbers

- Comparing but also reasoning: Which is better in this case?

Space Race: Number Choice

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Arithmetic Sequence

Encourage counting on from numeral
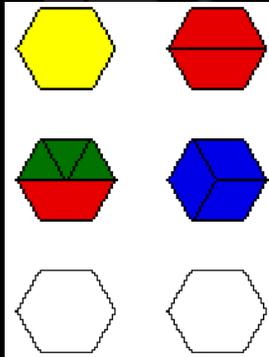Add numerals

Addition "choice" game

Two-digit addition

Figure the Fact: Two-Digit Adding
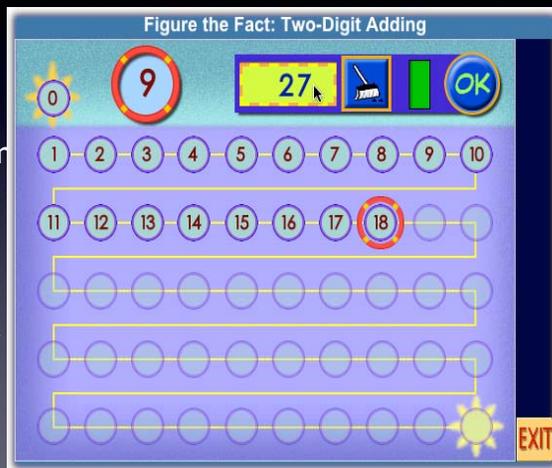
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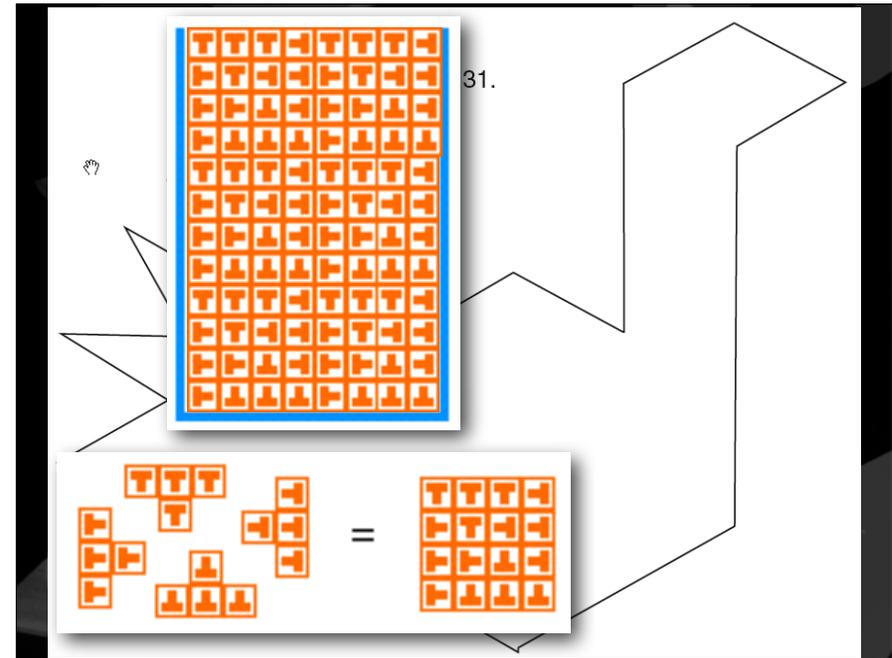# A Trajectory for Composing Geometric Shapes

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Substitution Composer

- Finds different ways to fill a frame, emphasizing substitution relationships.

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We Lay Groundwork Early…

- "First, I drew a triangle…

- then I had a trapezoid…

- then a parallelogram…

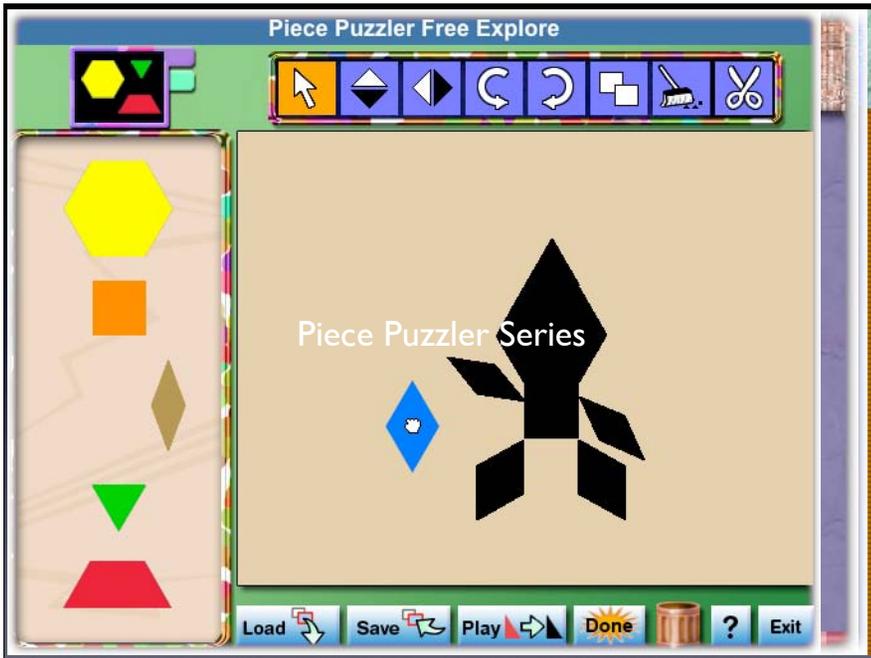- And when I was having hexagons,
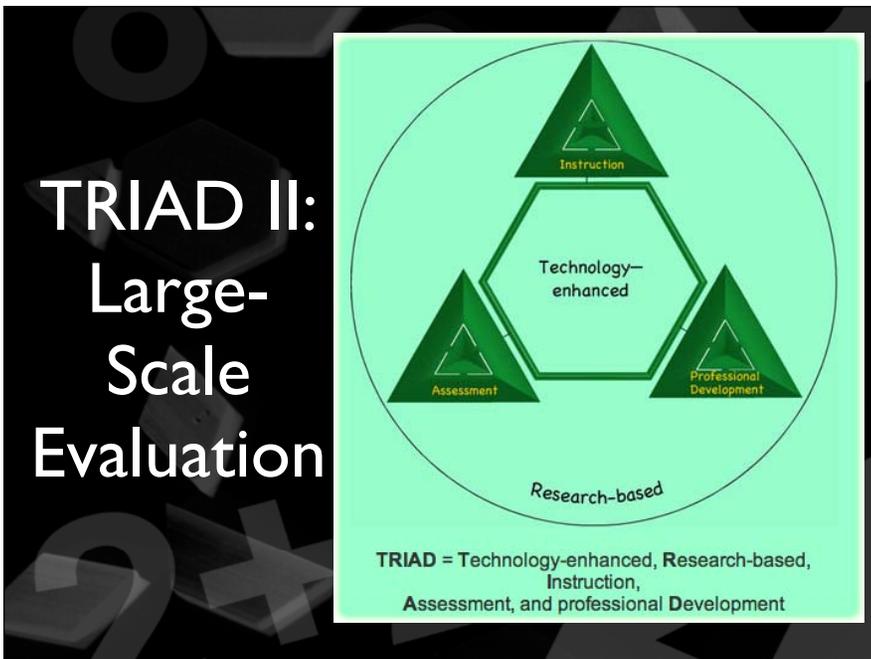
- I still had 10 triangles!"

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Create A Scene

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Piece Puzzler Series

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Building Blocks In the News

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TRIAD II: Large-Scale Evaluation

TRIAD = Technology-enhanced, Research-based, Instruction, Assessment, and professional Development

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Results: Child Assessment

- F(1, 32) = 40.52. p = .000+
- T Scores:
  - 50 Mean
  - 10 SD

Clements, D. H., & Sarama, J. (2008). Experimental evaluation of the effects of a research-based preschool mathematics curriculum. *American Educational Research Journal, 45*, 443-494.

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TRIAD

Control

Rasch scores

$p < .0001$

1.5

0.75

0

Pre   Post

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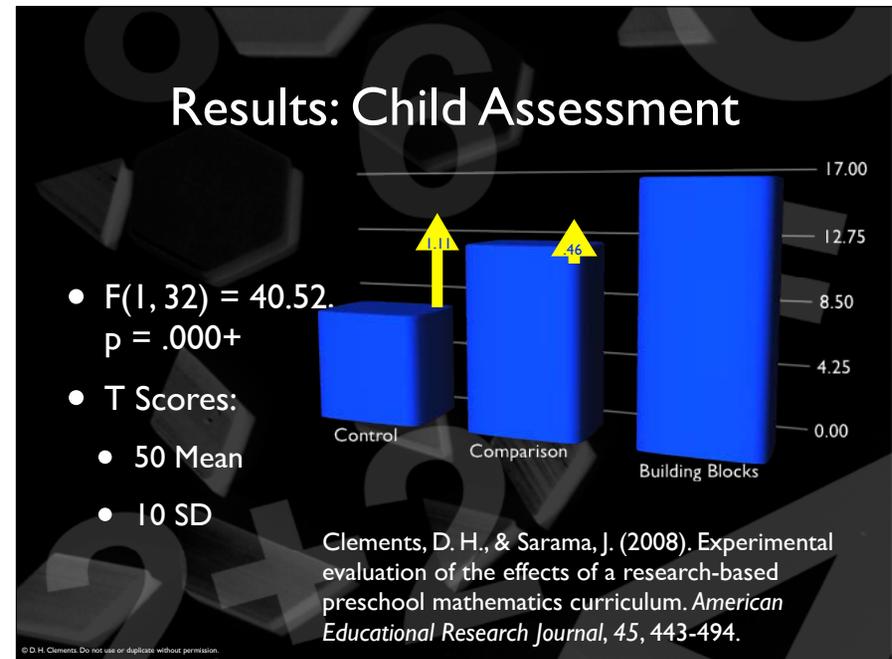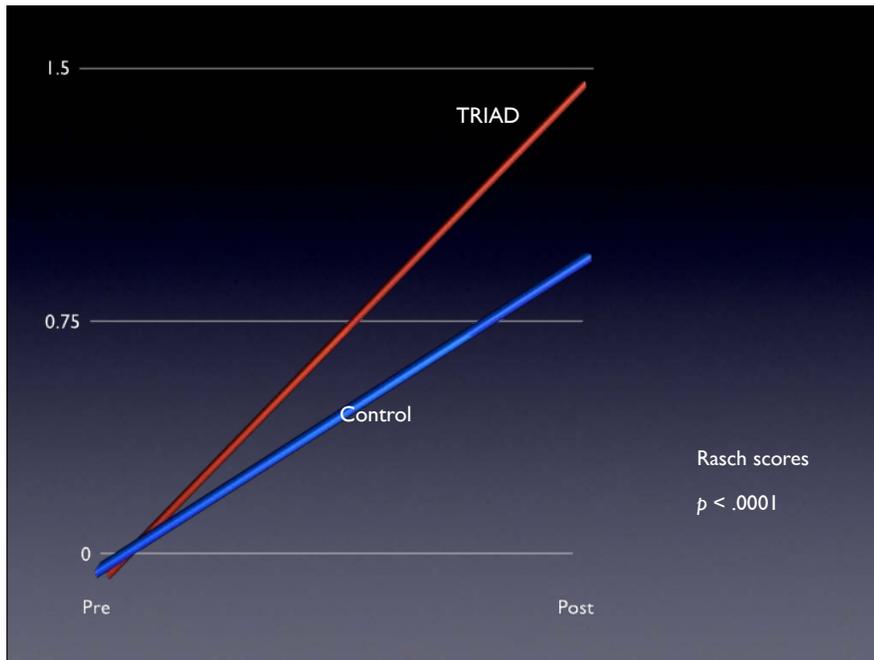
Using the Learning Trajectories

It takes time. A teacher talks about interviewing a child for report cards:

"She was able to do verbal counting to 8, and then when she slowed down, she could get to 11. So I said, "Can you make me a group of 6?" And so she did. So then I added, I did 12, I think. She couldn't do it.

Then I noted that, *so now I'm thinking in the trajectories*, I think she's a "Counter (Small Numbers)," right? She's on her way to being a "Counter (10)." She's in between the two. So that's what I was thinking of as I did this."
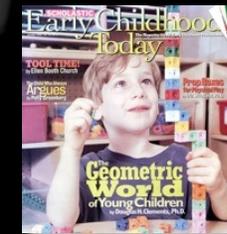
—Pat, 2004

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Early Childhood Mathematics Education Research
Learning Trajectories for Young Children
Julie Sarama & Douglas H. Clements

Learning and Teaching Early Math
The Learning Trajectories Approach
Douglas H. Clements & Julie Sarama

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Web Sites (and article download)

TRIADscaleup.org

BuildingBlocksMath.org

"If we teach today as we taught yesterday, we rob our children of tomorrow." –John Dewey

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